Make L the sum of the values along each side of the pentagon.

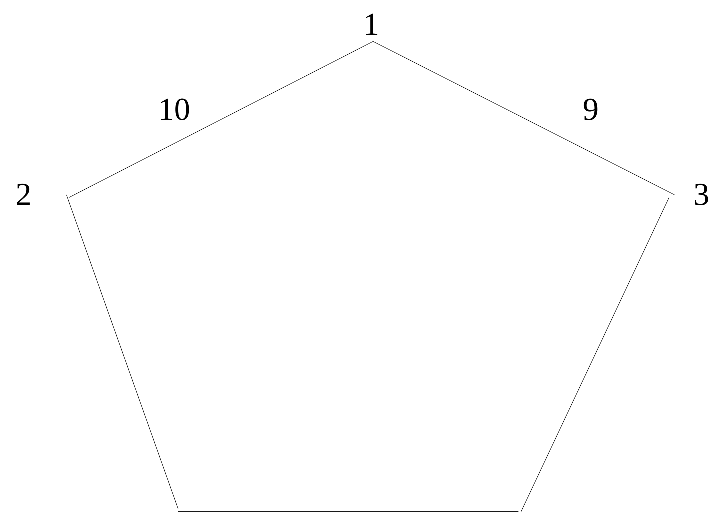
L cannot be less than 13 because there must be at least one occurrence of 10 in the pentagon, and $1+2$ are the least addends that could be combined with 10.

If $L=13$ then all the possible sets of addends of L are

$$10+2+1,\ 9+3+1,\ 8+4+1,\ 8+3+2,\ 7+5+1,\ 7+4+2,\ 6+5+2,\ 6+4+3$$

This list is easily found by systematic reduction of one addend and increase of the next by unity, until one of the rules defining sides is broken. Associativity applies to the addends in the list.

The integers 9 and 10 occur in only one set of addends each. Therefore, they must not be at a vertex, because numbers occurring at a vertex are in two sets of addends, one from each side that meets at the vertex. Both the set including 9 and the set including 10 include the integer 1, which may only appear once in the pentagon, so we know the two sets are adjacent about a vertex assigned the value one:



The side opposite 3's vertex from nine must be assigned a set of three addends that include 3 but no other used integers. This set must be $6+4+3$. Neither 6 nor 4 appears in any other sets that do not include already assigned integers, so neither can be at a vertex.

There can be no complete pentagons where $L=13$.

There exists at least one pentagon for $L = 14$, so that must be the least sum L.