Given

1. $f(x)=r+s+t+u$

2. $5r+4s+3t+6u=100$

3. $r\geq s\geq t\geq u\geq 0$

$$r=20 \wedge s=0 \wedge t=0 \wedge u=0 \rightarrow f(x)=20$$

$s \vee t \vee u$ cannot be decreased because each would then violate inequalities given in statement **2**.

r cannot be decreased while $s=0 \wedge t=0 \wedge u=0 \rightarrow r=100/5$ by given statement **2**. If *s*, or *t* were increased, $f(x)$ would increase because the coefficient of the increased variable(s) would be greater than the coefficient of *r*. The coefficient of *u* can be considered less than the coefficient of *r* because in order to increase *u*, *s* and *t* would have to be increased by an equal amount due to given **3**.

$$5\geq 4$$

$$5\geq \frac{4+3}{2}$$

$$5\geq \frac{4+3+6}{3}$$

Therefore, the minimum of $f(x)$ is *20*.

A multi-variable expression of the first degree with positive coefficients is equal to a positive constant:

$$ax+by=c$$

The sum of the variables *m*:

$$x+y=m$$

$$ax+by+ab-ab=c+ab-ab$$

$$a(x-b)+b(y+a)=c$$

$$x-b+y+a=m+a-b$$

$$a\geq b \rightarrow a-b\geq 0 \rightarrow m+a+b\geq m$$

Shows that when a multi-variable expression of the first degree with positive coefficients is equal to a positive constant, the sum of the variables is greater when a variable with a smaller coefficient is increased and a variable with a larger coefficient is decreased. Therefore, to get the maximum of $f(x)=r+s+t+u$ for $5r+4s+3t+6u=100$, the variables should be increased in ascending order of coefficient: *t*, *s*, *r*, *u*. Since $r\geq s\geq t\geq u\geq 0$, when $f(x)$ is at it's maximum, $r=s=t$. If *u* is decreased as much as possible because it has the largest coefficient, it must equal *0*.

$$5r+4s+3t+6\cdot 0=100$$
$$(5+4+3)r=100$$
$$r=s=t=\frac{100}{12}$$
$$\frac{100}{12}+\frac{100}{12}+\frac{100}{12}+0=25$$

The maximum of $f(x)$ is *25*.
The domain of $f(x)$ is $[20,25]$.