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If $x \wedge y$ are three digits, $(x||y) = 1000x + y$ because $1000x$ is the concantation of x and 000.

$$6(1000x + y) = 1000y + x$$
$$5999x = 994y$$
$$\frac{7}{7} \cdot \frac{857}{142} x = y$$
$$x = 142 \wedge y = 857$$

Similarly, if $x \wedge y$ are nine digits, $1000000000x$ is the concantation of x and 000000000 so that $(x||y) = 1000000000x + y$.

$$6(1000000000x + y) = 1000000000y + x$$
$$5999999999x = 999999996y$$
$$\frac{7}{7} \cdot \frac{857142857}{142857142} x = y$$
$$x = 142857142 \wedge y = 857142857$$

The sides $\overline{AB}$ and $\overline{BC}$ are equal. The remaining side is $\overline{AC}$. P is the perimiter and A is the area. The altitude streaches from B to the midpoint of $\overline{AC}$, which we will call D.

$$P = 2\overline{AB} + \overline{AC}$$

Pythagorean Theorem

$$\overline{BD} = \sqrt{\overline{AB}^2 - \left(\frac{\overline{AC}}{2}\right)^2}$$

$$A = \frac{\frac{\overline{AC}}{2}\sqrt{\overline{AB}^2 - \frac{\overline{AC}^2}{4}}}{2}$$

$$6(2\overline{AB} + \overline{AC}) = \frac{\frac{\overline{AC}}{2}\sqrt{\overline{AB}^2 - \frac{\overline{AC}^2}{4}}}{2}$$

$$48\overline{AB} + 24\overline{AC} = \overline{AC}\sqrt{\overline{AB}^2 - \frac{\overline{AC}^2}{4}}$$

$$48\frac{\overline{AB}}{\overline{AC}} + 24 = \sqrt{\overline{AB}^2 - \frac{\overline{AC}^2}{4}}$$

$$2304\frac{\overline{AB}^2}{\overline{AC}^2} + 2304\frac{\overline{AB}}{\overline{AC}} + 576 = \overline{AB}^2 - \frac{\overline{AC}^2}{4}$$

$$9216\overline{AB}^2 + 9216\overline{AB}\cdot\overline{AC} + 2304\overline{AC}^2 - 4\overline{AB}^2\overline{AC}^2 + \overline{AC}^4 = 0$$

Solution 1: $\overline{AB} = \overline{BC} = 80 \wedge \overline{AC} = 96$
Solution 2: $\overline{AB} = \overline{BC} = 85 \wedge \overline{AC} = 80$
Solution 3: $\overline{AB} = \overline{BC} = 90 \wedge \overline{AC} = 144$
Solution 4: $\overline{AB} = \overline{BC} = 130 \wedge \overline{AC} = 240$
Solution 5: $\overline{AB} = \overline{BC} = 175 \wedge \overline{AC} = 336$

I used an automated brute force aproach that found 87129 solutions with 5 digits or fewer. Some of these might be incorrect due to overflow of the long int data type, but I think that is unlikely because the unsigned long int produced the same result.

```
//C++ program for USAMTS year 16 round 3 problem 2
#include <iostream>
using namespace std;
int main(){
    long int countx=0;//length of AB
    long int county=0;//lenght of AC
    long int countsolutions=0;//number of solutions
    while(countx<100000){
```

```
        county=0;//reset segment AC length
        while(county<100000){
            if(9216*countx*countx+9216*countx*county+2304*county*county-4*countx*
countx*county*county+county*county*county*county==0){
//test for a solution without math library’s “pwr();” function.
                cout<<“AB= ”<<countx<<“ and AC=”<<county<<endl;
                countsolutions++;//increment AC length
            }
            county++;//increment AC length
        }
        if(countx%1000==0)
            cout<<“AB= ”<<countx<<“ has been checked.”<<endl;//output progress
        countx++;
    }
    cout<<“Done! Found ”<<countsolutions<<“ solutions.”<<endl;
    return 0;
}
```

If multiplication or addition is performed on a positive integer s, the n^{th} digit from the right of the result is only effected by the addend or factor and the n or fewer places from the right of s. If the same addition and multiplication operations are performed repetatively, once the same integer has apeared twice in the ones place, a repeating series of ones places has been defined, because there are no places to the right of the ones place. If the last digit of $s_1 = 1$, and subsequent values of s end in 4, 3, 0, and 1, respectively, and the addend and factor used to find each new s stay the same, then s will only have a ones digit of 1, 4, 3, or 0, always in that order.

Consider any s_a that ends in the digits 44. The preceeding digits of s_a will be represented by an ellipsis and the immediately preceeding digit by r. An expression surrounded by brackets is a digit.

$$\ldots r44 = s_a$$

$$\ldots r44 \cdot 3 + 1 = \ldots \{3r + 1\}33 = s_{a+1}$$

s_{a+1} must end in the digits 33.

$$\ldots \{3r + 1\}33 \cdot 3 + 1 = \ldots \{9r + 4\}00 = s_{a+2}$$

s_{a+2} must end in the digits 00.

The same argurment in reverse leads to the converse.

$$s_a = ...44 \leftrightarrow s_{a+1} = ...33 \leftrightarrow s_{a+2} = ...00$$

This recursive sequence forms some interesting patterns in other bases as well. In binary, the first digit from the right alternates between 1 and 0. The second digit is always 0. The third is 1 twice, then 0 twice, starting with the second number. The fourth digit from the right is 1 four times, then 0 four times, starting with the third number. In trinary, all the digits are always 1, and a new place is added on each recursion.

Law of cosines:

$$(10\sqrt{3})^2 = 2(10\sqrt{2})^2 - 2(10\sqrt{2})^2 \cos(\angle BAC)$$

$$\frac{1}{4} = \cos(\angle BAC)$$

$$\angle BAC \approx 1.318$$

θ is the angle formed by the planes ABC and DEF. $5\sqrt{3}$ is the radius of the

$$(cos(\theta) + 5\sqrt{2})cos(sin(\theta) = 5\sqrt{3} - sin(90 - \theta)$$